Report on the benchmarking of the event generator for fusion-evaporation reactions

The main aim of this project is the creation of the module of the GEANT4 platform for the description of the fusion-evaporation reaction.

The first part of the project contains the correct description of the fusion processes. The simulation of the fusion cross section was done based on the formulation Galster et al. HMI Ann. Rep. 1983, which have been later verified by A. Maj.

The next step was to test the evaporation part.

Complete, incomplete fusion

The starting point is the beam of a given energy, which pass by the target and produce the excited complex system – a compound nucleus. The fusion cross section depends on the kinetic energy of the mass and charge of the projectile and the target material, what influence on the energy losses in the material and change the kinetic energy of the projectile.

The fusion – evaporation reaction is characterized by the complete and incomplete fusion cross section and the maximal spin of the compound nucleus. The input values are mass and charge projectile and target and excitation energy in laboratory frame.
The derivation for the complete, incomplete fusion are taken from Galster et al. HMI Ann. Rep. 1983 and Wong, Phys. Lett B 42 182 (1972), Phys Rev. Lett. 31,766 (1973).

GEANT4 results

The GEANT4 simulations combine new modulo of the fusion cross section with existing packages calculating energy loses in the target.

Two option are available:
a) the constant fusion cross-section in the target: assuming the constant fusion cross section in all target deepness, we neglect the decreasing the kinetic energy in the target;
b) the energy losses decrease the fusion cross section but the calculation are time- consuming.

The simulation have been done for the reaction: ^{48}Ti + ^{40}Ca @ 300 MeV and the three target thicknesses: 1, 10 and 100 mg/cm^2 (Fig.1).

a) Assuming the constant fusion cross-section obtained from the derivations, we can investigate the kinetic energy of the merged system in the function of the target thickness and E_k of the compound nucleus leaving the target. The kinetic energy of the compound nucleus leaving the target is already corrected by the energy losses.

The main interest brings the Fig.1, where the fusion cross section is shown in the function of the target thickness (z). The influence of the energy loss in the target is clearly visible.

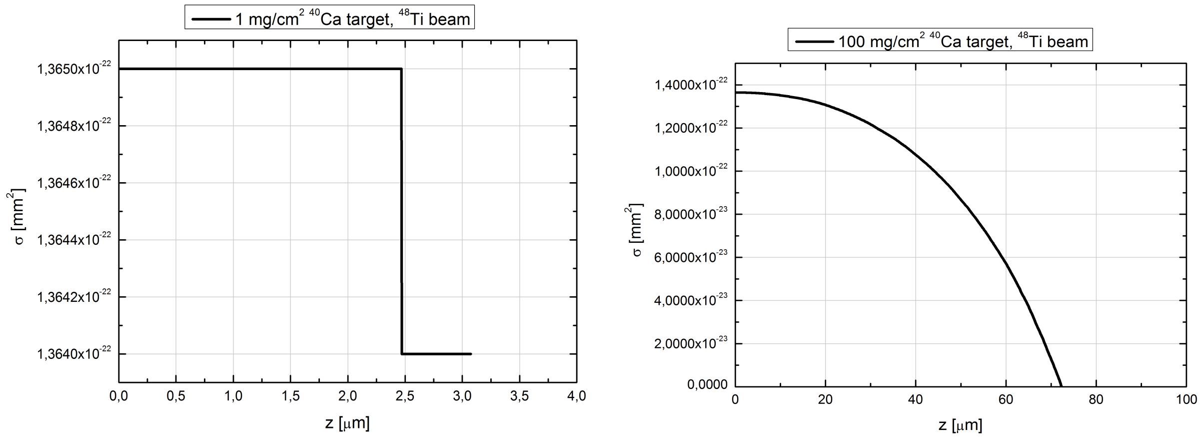


Fig. 1 Change of cross-section in the target (1 and 100mg/cm^2) due to loss of the projectile energy.

b) Taking into account the changes of the fusion cross – section we can say for the thick target (100 mg/cm^2) all compound nuclei are stopped in the target.

Remarks

1. The kinetic energy after fusion is independent on the constant or variant fusion cross-section option only for very thin targets.
2. The distribution of the kinetic energy of nuclei leaving target is more spread out for the option with inconstant cross-section.
3. The range of the excitation energy is similar but the structure is different.
4. For the thick target, when the compound is stopped in the material the excitation energy range is wider for the constant cross section option but it is unimportant as far as the compound nucleus is stopped in the target material.

The next steps are the calculations of the transmission coefficient in GEANT4, which are in the newest version. To compare the results with the original version of the Monte Carlo Cascade, we have to include option in GEANT4 with the old optical potential parameters or change it in our Fortran codes.

Transmission coefficients

The code which was done up to now, for the fusion cross-section calculations omits the problem of the tunnelling by the coulomb barrier. It always gives 0 when the excitation energy was lower than the Coulomb barrier for the distance between two nuclei R_f. New version contains the possibility of calculating the Bass and Proximity potentials. Using the Hill – Wheller formula (HW) it is possible to estimate the empirical transmission coefficients and calculate the fusion cross-section. The Hill – Wheller formula is taken from D.L. Hill, J.A. Wheller, Phys.Rev. 89 (1953)1102). The transmission coefficients are calculated for given potential. We tested the Proximity potential (J. Blocki, Ann. Phys. 105, 427 (1977)) and Bass potential (R. Bass Nucl. Phys. A231 (1974)45), which give us also the angular momentum.

Technical details and results

The test for the transmission coefficient starts with the plotting various potentials. The verification is done by an comparison with the data collected within the web page: http://nrv.jinr.ru/nrv/webnrv/fusion1. The results for the fusion cross-section obtained with the proximity potential and by the GEMINI++ are shown below (Fig.2). The discrepancy for the lower $1/E_{cm}$ reactions are due to the omitting the deformation of the nucleus during the calculations the transmission coefficients.

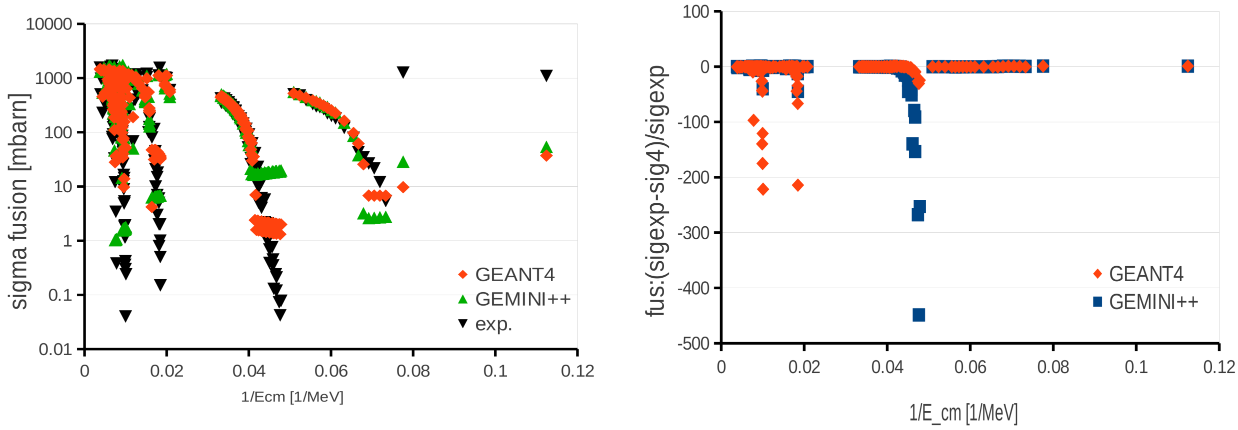


Fig. 2 (Left) The fusion cross-section calculated with the proximity potential (GEANT4), by the GEMINI++ and compared with the experimental data for 250 reactions. (Right) The correctness of the fusion cross-section ($sigma_{theo}$-$sigma_{exp}$)/$sigma_{exp}$) calculated with the proximity potential ($sigma_{theo}$=$sigma_{prox}$ "GEANT4"), by the GEMINI++ ($sigma_{theo}$=sigma"GEMINI++") for 250 reactions.

Weiskopf vs Hauser-Feschbach evaporation model

The evaporation part, which existed in the GEANT4 up to now, contained only Weiskopf model. The differences between our theoretical calculations done by statistical code GEMINI++ and dynamical code with Langevin equations contains the Hauser – Feschbach approach in the evaporation part and the model for yrast line (minimal energy of excited nuclei at given angular momentum) are shown in Fig.3.

The main input parameters for the evaporation part of the GEANT4 class are the Z,N of the target and the projectile and the beam energy. This parameters are used to calculate the fusion cross section, as it has been discussed above and the critical spin for the compound nucleus - using the Bass prescription. The triangular distribution of the available values for the spin is assumed. The Monte Carlo GEANT4 class is used to randomly choose the values of the spin from those distribution. The energy/spin phase space is limited by the *yrast* line, obtained from the minimisation of the Lublin – Strasbourg Drop (LSD) formula (K. Pomorski, J. Dudek, Phys. Rev. **C 67** (2003) 044316.) in the multidimensional deformation space and collected as the GEANT4 library for all nuclei within Z=16-130 range. The randomly chosen sort of particle emitted during the cascade contains the neutrons, protons, alphas, deuterons and gamma rays.

The output observable are: the energy spectra and multiplicities of the emitted particles, the mass and charge distribution of the evaporation residues, the mass/charge and the excitation energy as well as the angular momentum of the compound nucleus.

The evaporation residues for reaction ^{27}Al(E_{lab}=490 MeV)+^{84}Kr leading to ^{111}In (W. Schneider et al., Nucl. Phys. **A371** (1981) 493) were compared with our GEANT4 class calculations and GEMINI++ results (Fig.3, left) also for reaction as above: ^{48}Ti(E_{lab}=300 MeV)+^{40}Ca leading to ^{88}Mo. We overestimate the lighter residues.

The new GEANT4 class has been prepared to calculate the evaporation with the Hauser – Feschbach method. The main advantage is possibility to control of the spin of the evaporation residues. The Figure 3 shows the mass and charge distribution of the residues for ^{48}Ti(E_{lab}=300 MeV)+^{40}Ca leading to ^{88}Mo (right) and present results for reaction ^{27}Al(E_{lab}=490 MeV) +^{84}Kr leading to ^{111}In (left). There are also tests for different y*rast* lines: *yrast* line for spherical shapes and for deformed shapes obtained from the RLSD (LSD model with the rigid-body rotational energy). The best agreement is for the RLSD *yrast* line.

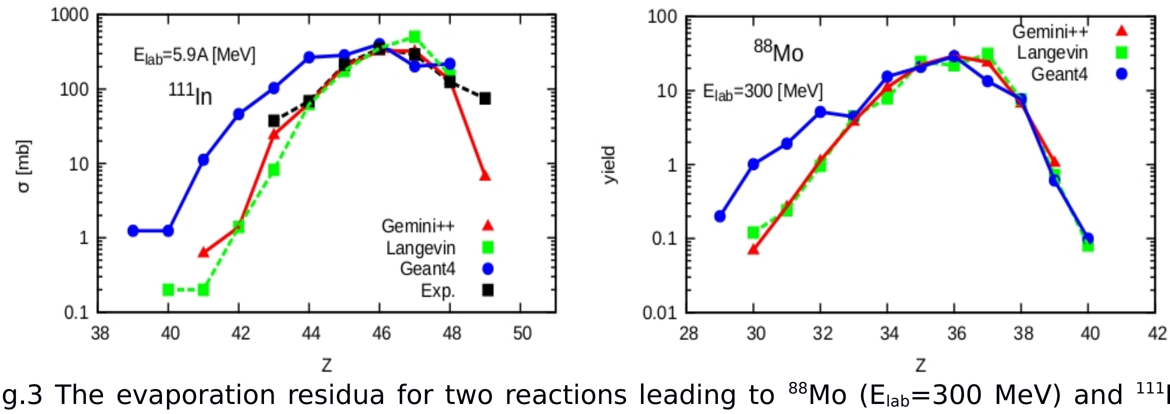


Fig.3 The evaporation residua for two reactions leading to ^{88}Mo (E_{lab}=300 MeV) and ^{111}In (E_{lab}= 490MeV).

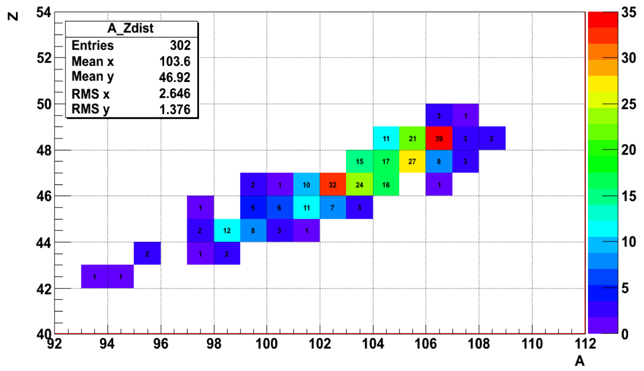
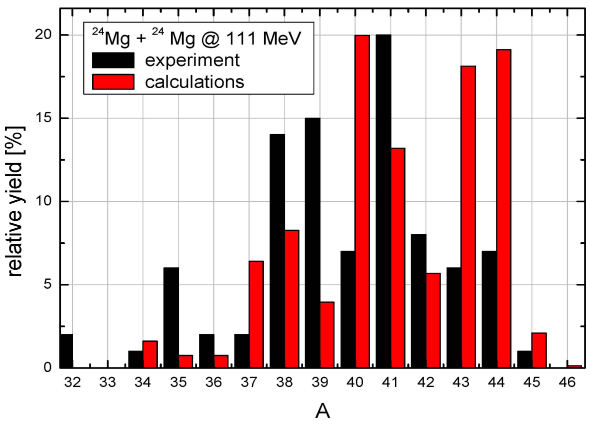
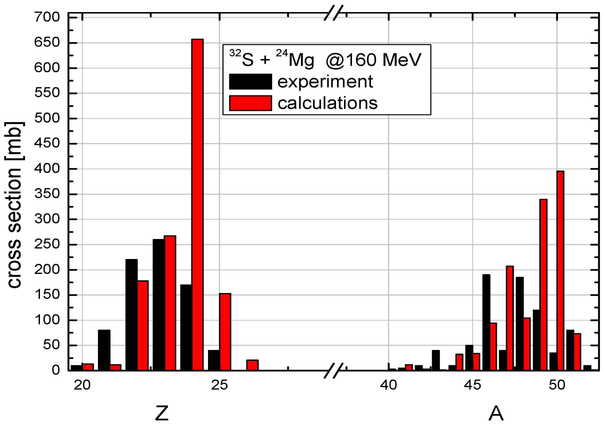


Fig. 4 The evaporation residua map for reaction leading to ^{111}In (E_{lab}=490 MeV)

The other reactions have been calculate to test if the similar agreement is obtained in other mass ranges. The reaction, for which experimental data exist in the literature, have been chosen to give variety of compound masses. Figure 5(left) illustrate the relative cross section for reaction ^{24}Mg+^{24}Mg (F. W. Prosser et al. Phys. Rev. **C 40**, 2600 (1989)) for E_{lab}=111 MeV (4.625 MeV/A) and the mass distribution has been reproduced by the GEANT4 calculations. This reaction is a typical fusion – evaporation process because the excitation energy is too low to open fission channel.

Next reaction, presented in Fig.5(right) is ^{32}S+^{24}Mg (F. Pulhofer and W. F. W. Schneider Phys. Rev. **C 16**, 1010 (1977)) with the E_{lab}=160.7 MeV (5 MeV/A). The test was done simultaneously for the cross section and the mass/charge residues distribution. The comparison is very accurate.





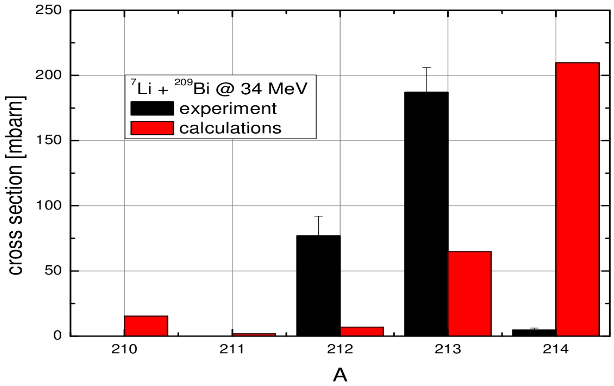


Fig. 5 The evaporation residua for reactions leading to ^{48}Cr, ^{56}Ni and ^{216}Rn. Comparison of the experimental mass/charge distribution and GEANT4 calculations with the RLDM yrast line.

The last reaction in Fig. 5(bottom) is ^{7}Li (E_{lab}=34 MeV (4.9 MeV/A))+^{209}Bi which produces heavy compound nucleus ^{216}Rn and only few residues are obtained due to very low excitation energy. In this case the agreement is a little bit worse than in previous case but for such heavy system the fission process is very important.

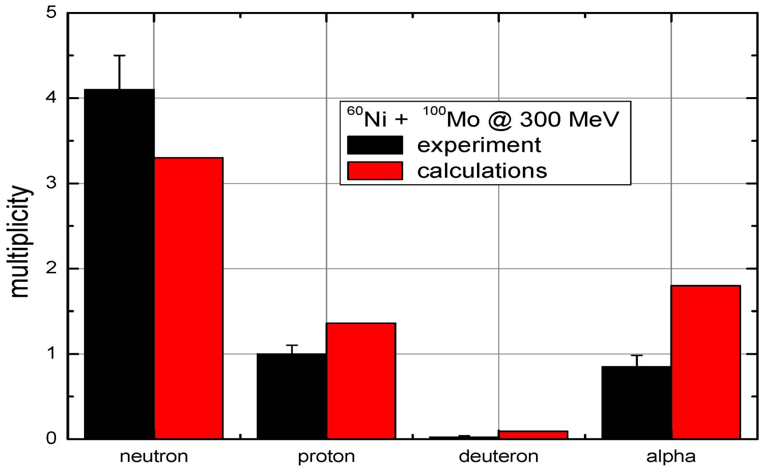


Fig. 6 The neutron, proton, deuteron and alpha particle multiplicities evaporated in reaction ^{60}Ni (E_{lab}=300 MeV)+ ^{100}Mo compared to GEANT4 calculations.

The Figure 6 illustrate the reproduction of the evaporated particle multiplicities for various spices in reaction ^{60}Ni (E_{lab}=300 MeV)+ ^{100}Mo (Phys. Rev. **C 67**, 044611, 2003). It seems than the neutron emission is underestimated and the alpha multiplicity is overestimated but in general the agreement is good.

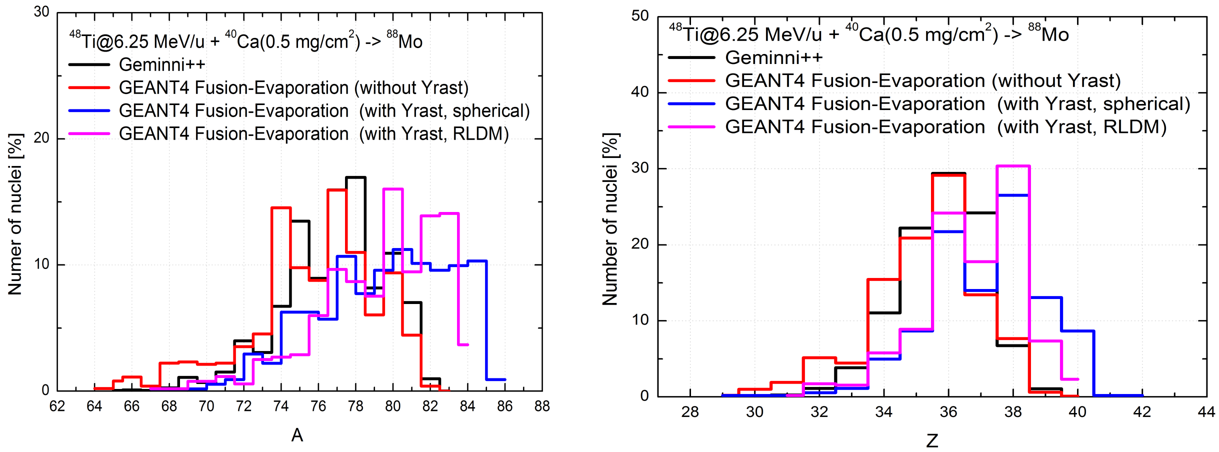


Fig. 7 The evaporation residua for reaction leading to ^{88}Mo (E_{lab}=300MeV).

The next step was to verify the model for *yrast* line. The library with *yrast* line calculation has been done with rotating LSD formula and compared to the widely used Finite Range Liquid Drop Model (FRLDM) *yrast* lines (A. Sierk, Phys. Rev. **C 33**, 2039 (1986)). Figures 7 and 8 present the mass and charge distribution of the residues for the above reaction obtained with our new GEANT4 class and RLSD yrast line obtained from excited ^{111}In and ^{88}Mo compound nucleus.

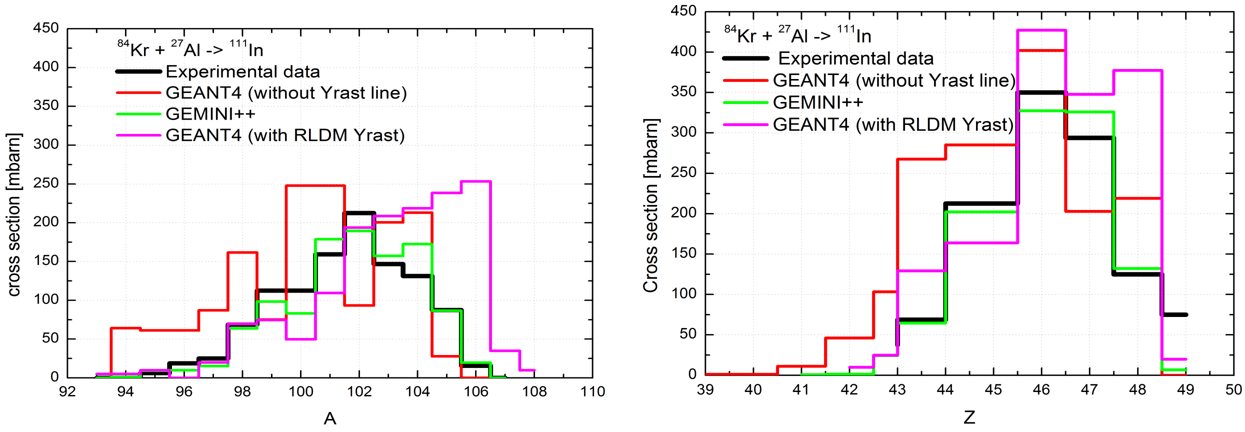


Fig.8 The evaporation residua for reaction leading to ^{111}In (E_{lab}=490 MeV).

The Figure 9 illustrate the mass distribution obtained with two *yrast* lines.

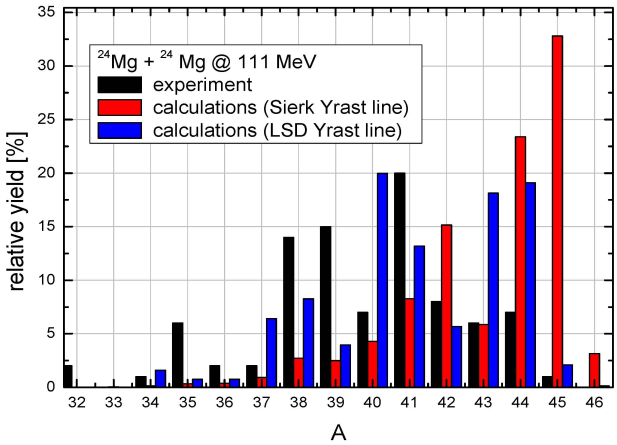


Fig.9 The evaporation residua for reactions leading to ^{48}Cr. Comparison of the experimental mass distribution and GEANT4 calculations with the RLSD and FRLDM yrast line.

The FRLDM model gives too little yield for the light residues and too high for heavy evaporation residues. This behaviour is confirmed when the multiplicity of evaporated neutrons is smaller then the experimental one and the results coming from RLSD model.

Implementation in GEANT4

The code is divided into the main files:
G4FusionCrossSection - fusion cross section calculation code, used to get the probability for ***DiscreteProcess*** of fusion in GEANT4;
G4LEFussionModel - fusion model, in which excited compound nucleus is created, it provides possibility to assign the evaporation mechanism (by providing the evaporation class);

G4Evaporation - provides de-excitation of compound nucleus, by emissions of proton, neutron, deuteron, alpha particles and gamma-rays, and while using ***G4EvaporationDefaultGEMFactory*** also heavier nuclei (up to ^{28}Mg).

Summary:
1. The energy loss in the target is taken into account.
2. Reproduction of the fusion cross-section is very good.
3. The Weiskopf model of the evaporation has been tested.
4. The Hauser – Feschbach model was tested without any *yrast* line and with *yrast* line for spherical shapes and for the *yrast* line coming from the RLSD approach and FRLDM model.
5. The results has been compared with existing data and the prediction for ^{88}Mo has been compared with state-of-art statistical code GEMINI++.
6. New GEANT4 classes are compatible with the GEANT4 filter for various detector geometry classes, can be also combined with the class described fission process.