Super heavy nuclei

Status and difficulties of self-consistent mean-field approaches

Status and prospect for SHE

- Several studies in the last 10 years, self-consistent mean-field methods can be routinely applied to SHE.
- Short review of typical results
- Studies of very heavy nuclei = tests of models (interaction and method)?
- Main problems

HFB methods

- Effective interaction for mean-field and pairing:

Minimization of the total energy $\rightarrow$ HFB ground state $|\Phi\rangle$

$$E = \langle \Psi | \hat{H} | \Psi \rangle = E[\rho, \kappa, \kappa^*]$$

with constraints on N and Z $\langle \Psi | \hat{N}_q | \Psi \rangle = N_q$.

Solution of the HFB equations on a 3-dimensinal mesh (triaxiality included)

Minimization of $E^{\lambda} = E - \lambda_q \langle\Psi|\hat{N}_q|\Psi\rangle$ in a basis

$$
\begin{aligned}
\mathcal{H}\begin{pmatrix} U_n \\ V_n \end{pmatrix} &= e_n \begin{pmatrix} U_n \\ V_n \end{pmatrix}, \\
\mathcal{H} &= \begin{pmatrix} h-\lambda & \Delta \\ -\Delta^* & -h^*+\lambda \end{pmatrix},
\end{aligned}
$$

U and V are transformation matrices

e_n are the quasi particle energies

Great similarities between Skyrme, RMF and Gogny studies.

Same kind of successes, and of problems.

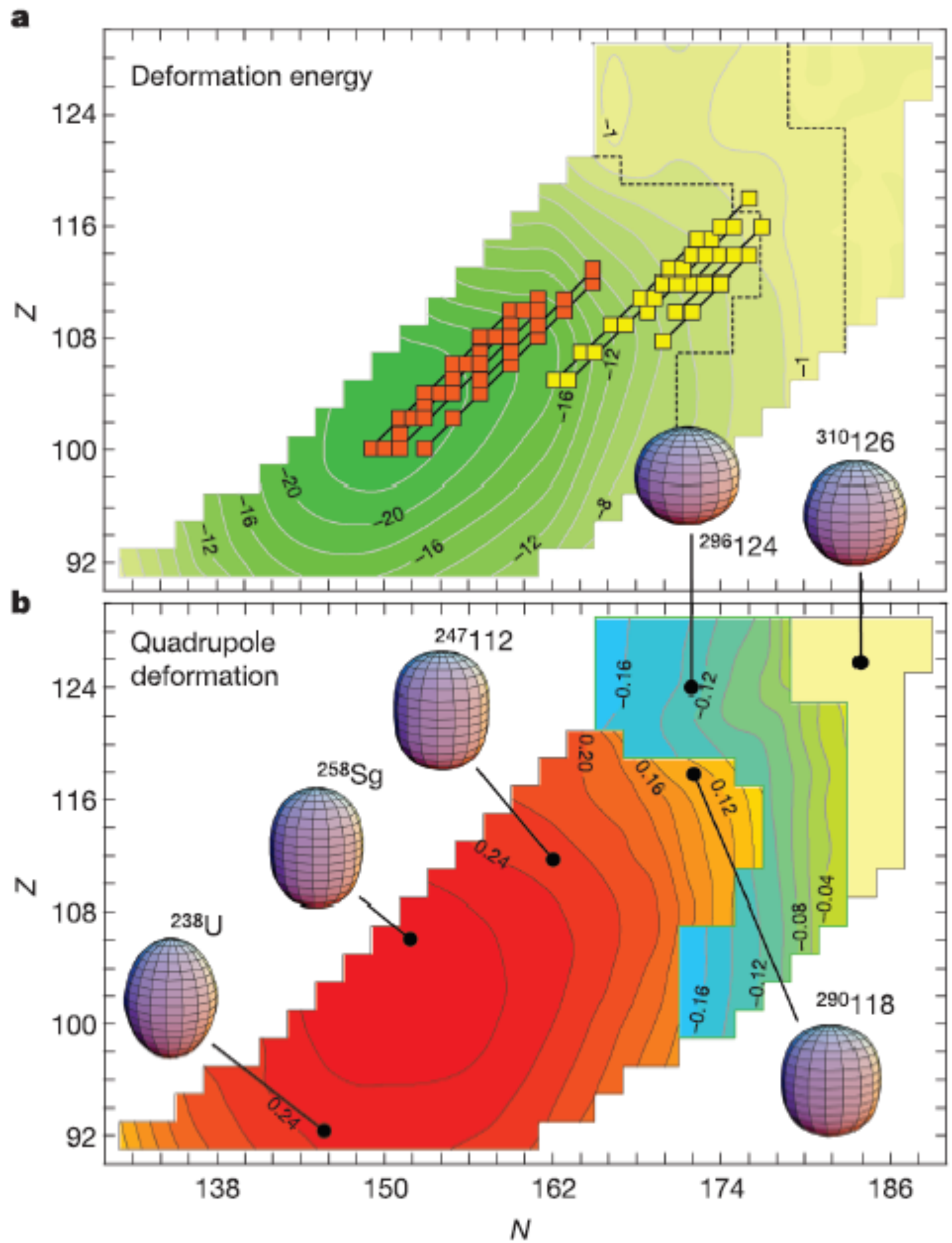
Main reference for this talk:
S.Cwiok , PHH and W. Nazarewicz PRL 83 1108 1999
Nature 433 705 2005
P. Bonche, M.Bender and PHH PRC 70 54304 2004
A.Afanasjiev et al. J. Phys.: Conf. Ser. 312 092004 2011
M. Bender et al. Phys. Rev C 60 034304 1999

A. Sobiczewski and K. Pomorski, Prog in Part. Nucl. Phys. 58 292 2007

Collaboration also with V. Hellemans and K. Washiama

Skyrme HFB

Deformation properties of super-heavies



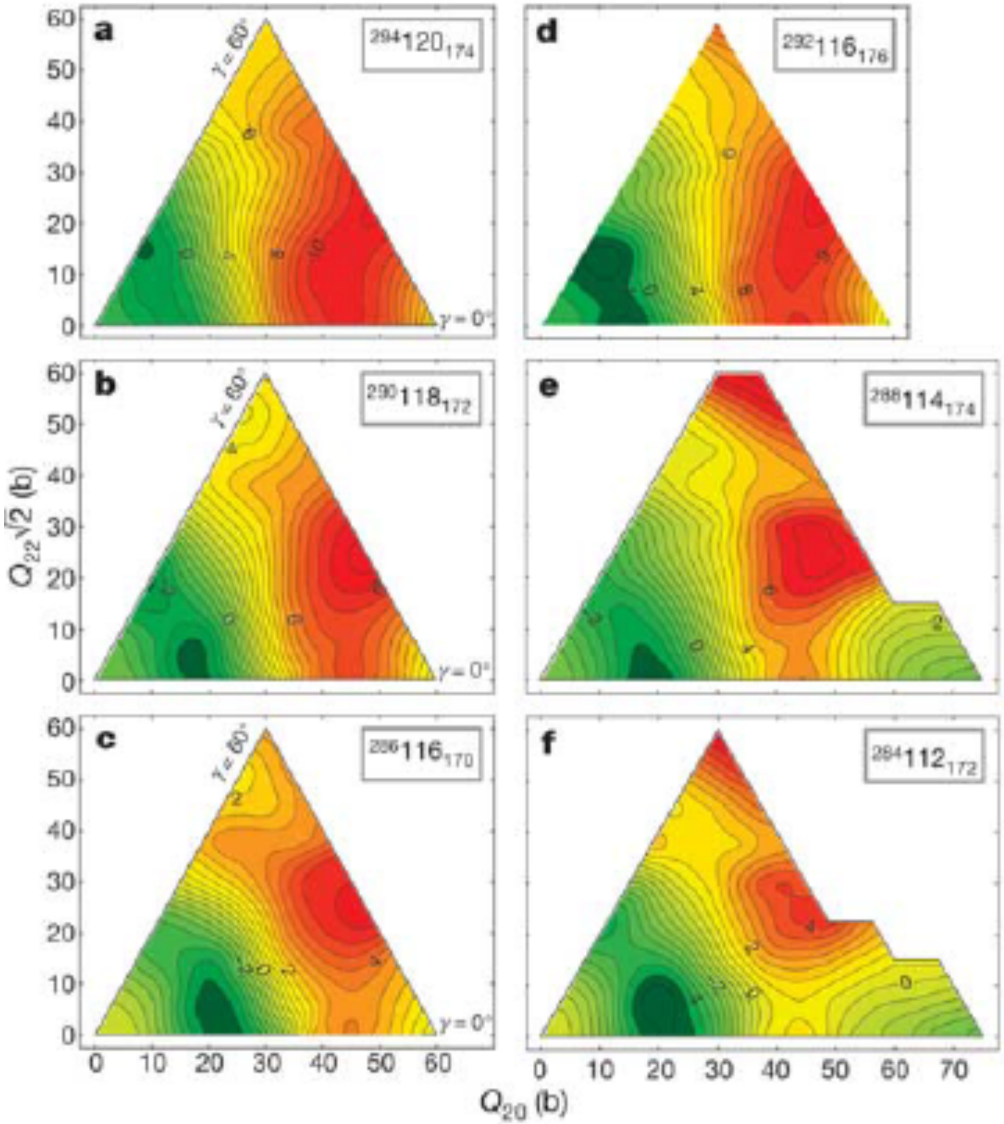


Figure 3 Potential energy surfaces of the members of the α-decay chains of $^{294}120$ (**a–c**) and $^{292}116$ (**d–f**) in the (Q_{20}, Q_{22}) plane calculated with the SLy4 energy density functional. It is seen that both α-decay sequences are associated with transition from oblate (or triaxial shapes) in the parent nuclei to prolate shapes in lighter daughter nuclei. The difference between contour lines is 0.5 MeV.

Microscopic calculations include automatically all the deformations that are not excluded by symmetry restrictions

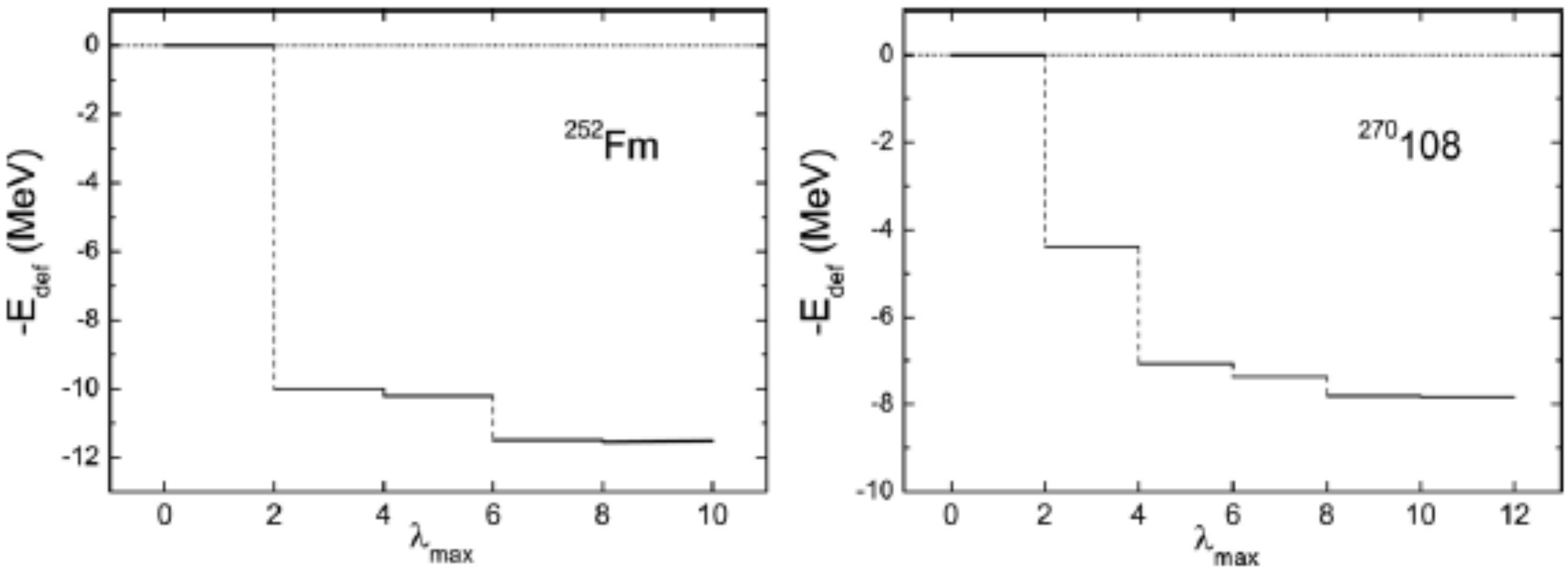
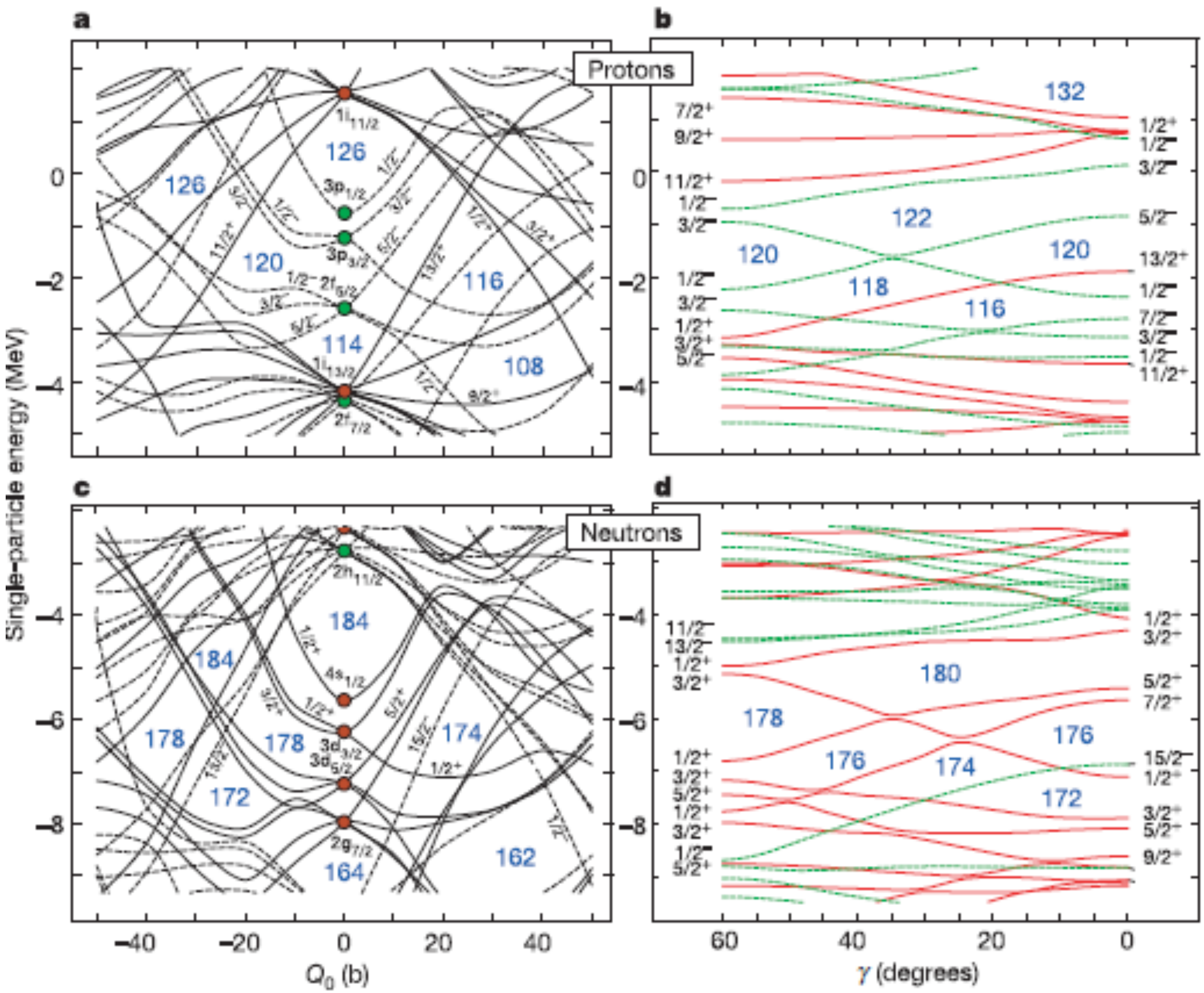


Fig. 9. Dependence of the deformation energy (taken with minus sign), $-E_{def}$, on λ_{max} for the nucleus ^{252}Fm (l.h.s) and ^{270}Hs (r.h.s.) [65].

A. Sobiczewski, K. Pomorski / Progress in Particle and Nuclear Physics 58 (2007) 292–349



a
b
c
d
Protons
Neutrons
Single-particle energy (MeV)
Q_0 (b)
γ (degrees)
126
126
120
116
114
108
132
122
120
120
118
116
184
184
178
178
172
174
164
162
180
178
176
176
174
172
1i11/2
3p1/2
3p3/2
2f5/2
1i13/2
2f7/2
2h11/2
4s1/2
3d3/2
3d5/2
2g7/2

Beyond ground state properties of even-even nuclei

Breaking of time reversal invariance

by a cranking constraint:

$H' = H - \omega J_x$ rotational bands for deformed nuclei

by quasi particle excitations:

Odd nuclei : 1 qp states: $\beta_i^\dagger |0\rangle$

Even nuclei: 2qp states

Still a mean-field method

Full self-consistency for mean-field and pairing

No direct relation with the single-particle spectrum.

One does not have: $E_{qp,i} = ((\varepsilon_\iota - \lambda)^2 + \Delta^2)^{1/2}$

How to test the models?

How reliable are the models for nuclei just below the SHE?

Tests on: deformation properties (and fission barriers)
- moment of inertia of rotational bands
- spectra of odd nuclei
- 2qp isomers in even nuclei

Nucleus	Orbital	Energy (MeV)	β_2
$^{293}116$	$[604]\frac{7}{2}^+$	0	0.09
(10.47)	$[602]\frac{5}{2}^+$	0.31	0.09
	$[611]\frac{1}{2}^+$	0.52	0.08
	$[707]\frac{15}{2}^-$	0.93	0.09
$^{289}114$	$[707]\frac{15}{2}^-$	0	0.12
(9.64)	$[611]\frac{1}{2}^+$	0.52	0.11
	$[604]\frac{7}{2}^+$	0.79	0.13
	$[602]\frac{5}{2}^+$	1.17	0.12
$^{285}112$	$[611]\frac{1}{2}^+$	0	0.14
(8.88)	$[611]\frac{3}{2}^+$	0.60	0.14
	$[707]\frac{15}{2}^-$	0.62	0.13
	$[606]\frac{11}{2}^+$	0.65	0.15
	$[604]\frac{9}{2}^+$	0.72	0.15
$^{281}110$	$[604]\frac{9}{2}^+$	0	0.19
(9.32)	$[606]\frac{11}{2}^+$	0.07	0.19
	$[611]\frac{1}{2}^+$	0.12	0.18
	$[611]\frac{3}{2}^+$	0.59	0.17
	$[613]\frac{5}{2}^+$	0.65	0.17
	$[716]\frac{13}{2}^-$	0.94	0.17
$^{277}108$	$[611]\frac{1}{2}^+$	0	0.21
	$[604]\frac{9}{2}^+$	0.04	0.20
	$[613]\frac{5}{2}^+$	0.31	0.21
	$[716]\frac{13}{2}^-$	0.36	0.21
	$[611]\frac{3}{2}^+$	0.38	0.20

Nucleus	Orbital	Energy (MeV)	β_2
$^{293}118$	$[707]\frac{15}{2}^-$	0	0.11
(11.59)	$[611]\frac{1}{2}^+$	0.16	0.10
	$[602]\frac{5}{2}^+$	0.84	0.12
	$[604]\frac{7}{2}^+$	0.98	0.10
$^{289}116$	$[611]\frac{1}{2}^+$	0	0.13
(10.18)	$[606]\frac{11}{2}^+$	0.62	0.14
	$[611]\frac{3}{2}^+$	0.66	0.13
	$[604]\frac{9}{2}^+$	0.69	0.14
	$[707]\frac{15}{2}^-$	0.72	0.13
$^{285}114$	$[606]\frac{11}{2}^+$	0	0.16
(10.60)	$[611]\frac{1}{2}^+$	0.04	0.16
	$[611]\frac{3}{2}^+$	0.15	0.16
	$[604]\frac{9}{2}^+$	0.16	0.16
	$[613]\frac{5}{2}^+$	0.28	0.15
	$[716]\frac{13}{2}^-$	0.73	0.16
$^{281}112$	$[611]\frac{1}{2}^+$	0	0.19
(10.85)	$[604]\frac{9}{2}^+$	0.07	0.19
	$[611]\frac{3}{2}^+$	0.37	0.19
	$[613]\frac{5}{2}^+$	0.41	0.19
	$[606]\frac{11}{2}^+$	0.42	0.19
	$[716]\frac{13}{2}^-$	0.51	0.19
$^{277}110$	$[716]\frac{13}{2}^-$	0	0.22
(11.07)	$[613]\frac{5}{2}^+$	0.02	0.21
	$[611]\frac{3}{2}^+$	0.07	0.21
	$[611]\frac{1}{2}^+$	0.15	0.22
	$[604]\frac{9}{2}^+$	0.27	0.21
	$[606]\frac{11}{2}^+$	0.83	0.21

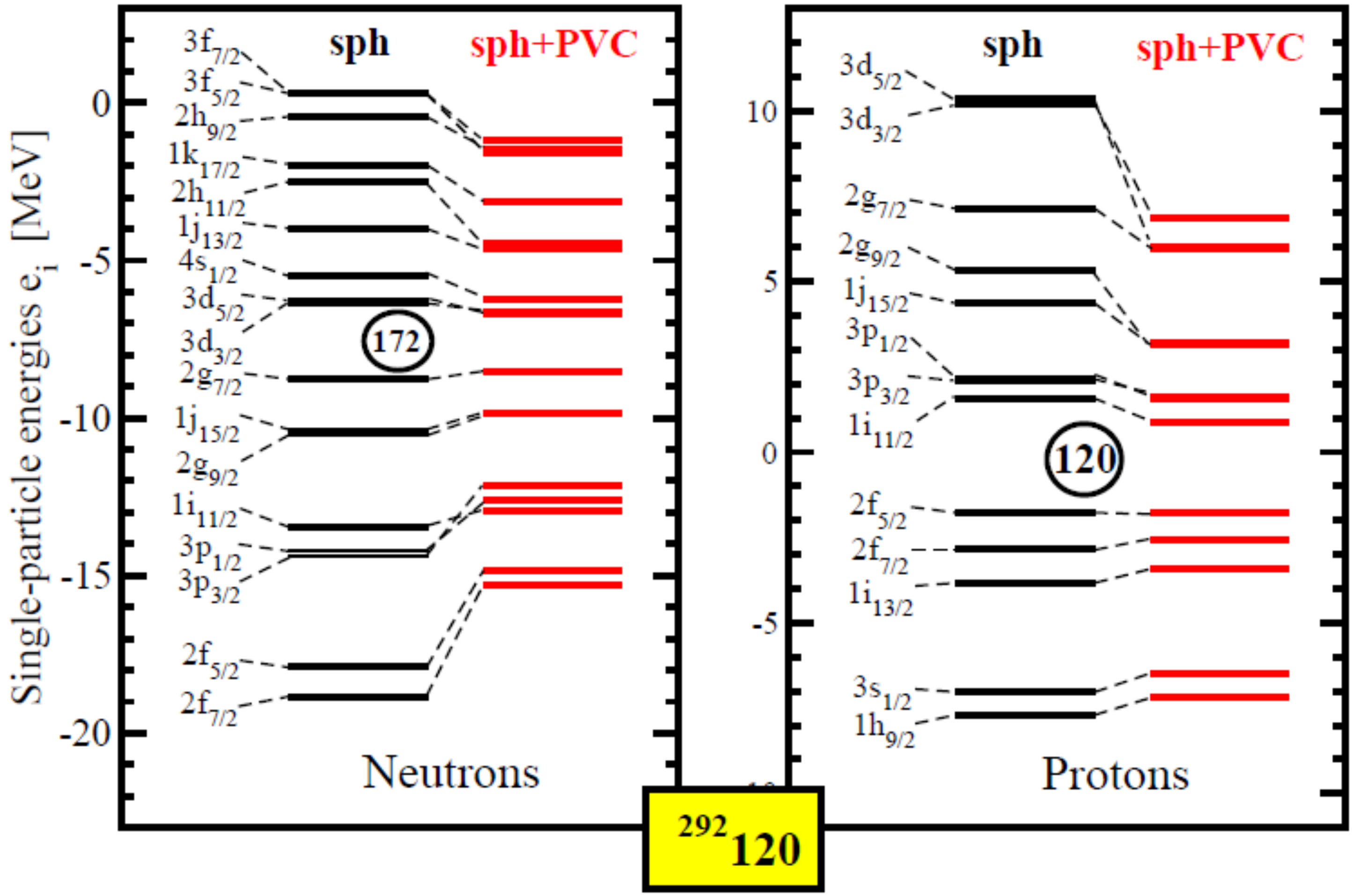


Figure 3. Neutron and proton single-particle states in the $^{292}120$ nucleus. The left colum each panel shows the spectra obtained in pure RMF calculations, while right column the spec computed within RMF with allowance for the particle-vibration coupling. The calculations performed at spherical shape employing the NL3* parameterization (from Ref. [15]).

A. Afanasjev, to published (from J. Phys.: Conf. Ser. 312 092004 (2011))

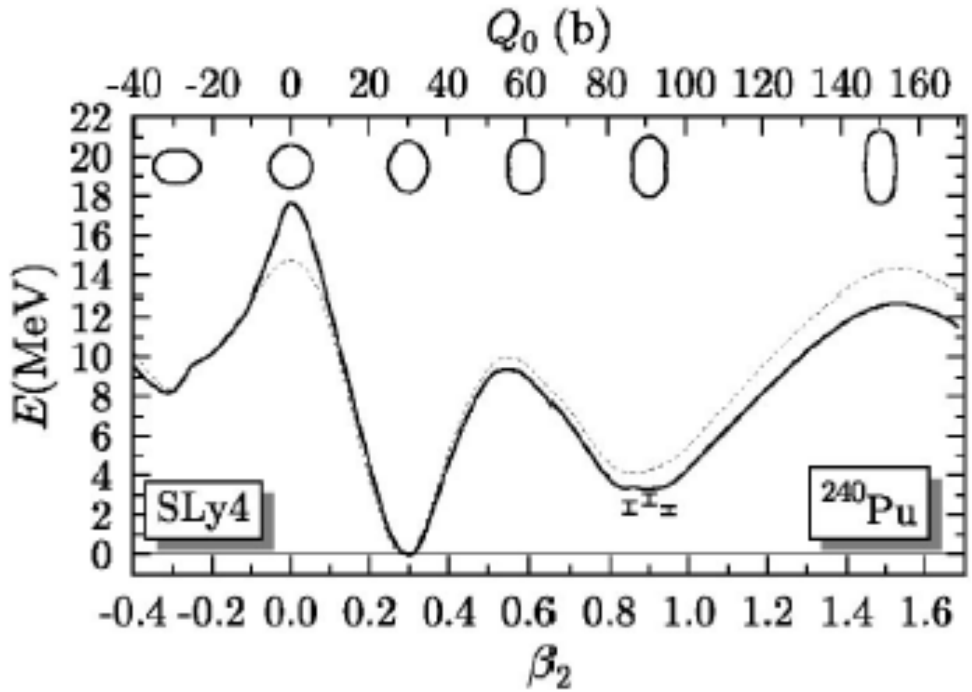


FIG. 1. Deformation energy curve of ^{240}Pu obtained with SLy4 projected on N and Z (dashed line) and projected on N, Z, and $J=0$ (solid line). All energies are normalized to the deformed ground-state value of each curve. The available experimental data for the excitation energy of the superdeformed band head are shown at arbitrary deformation (see text). Shapes along the path are indicated by the density contours at $\rho=0.07\ \mathrm{fm}^{-3}$.

Axial calculation, no octupoles but projection on N, Z and J=0
It decreases the excitation energy by around 3 MeV

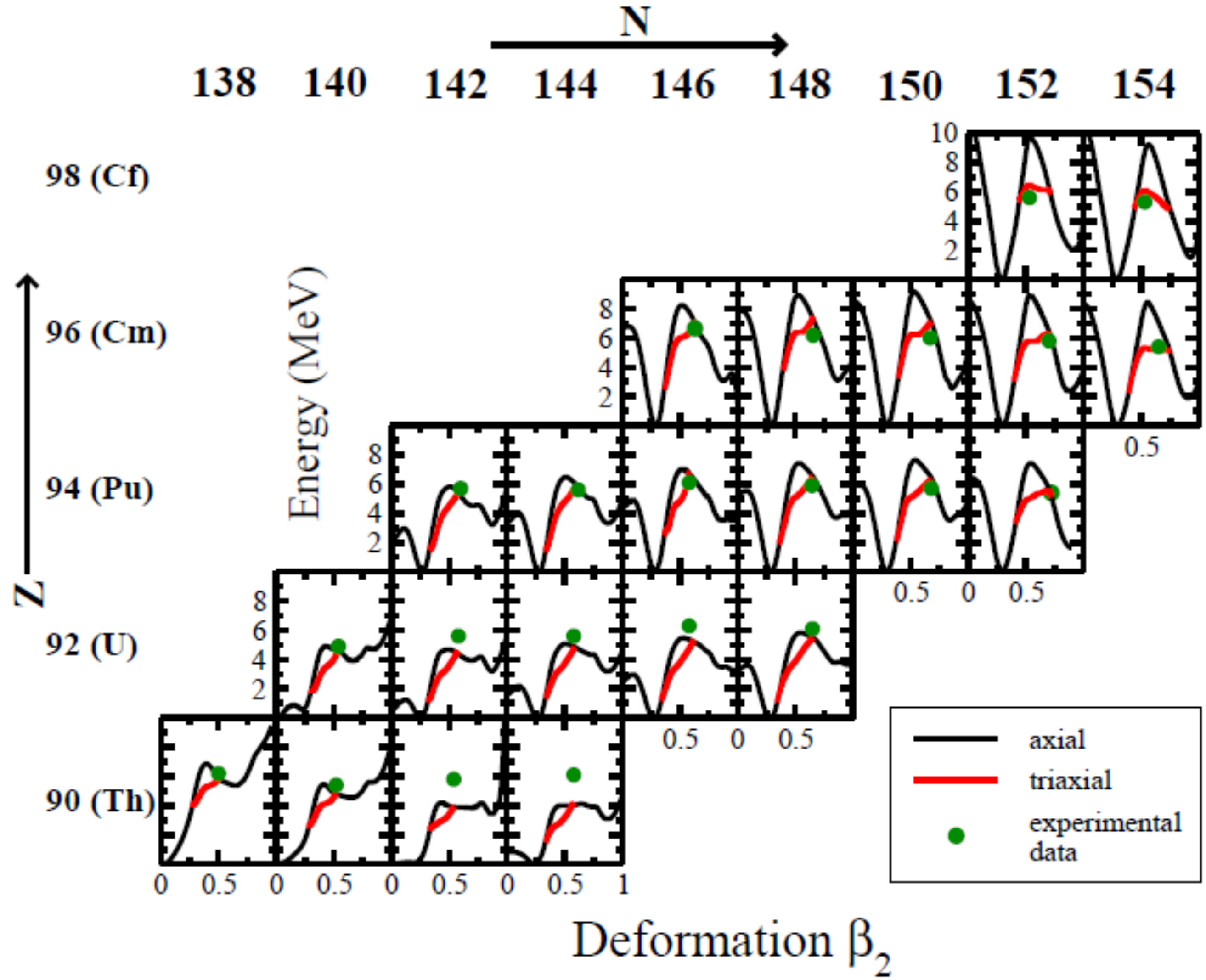


Figure 6. Deformation energy curves of even-even actinide nuclei obtained in RMF+BCS calculations with the NL3* parameterization. Experimental data are taken from Table IV in Ref. [42]. A typical uncertainty in the experimental values, as suggested by the differences among various compilations, is of the order of ±0.5 MeV [42]. The deformation parameters β and γ are determined using the expressions $\beta = \frac{4\pi}{3AR_o^2}\sqrt{\langle Q_{20}\rangle + 2\,\langle Q_{22}\rangle}$ with $R_0 = 1.2A^{1/3}$ and $\tan\gamma = \frac{\sqrt{2}\langle Q_{22}\rangle}{\langle Q_{20}\rangle}$ [41].

Moments of inertia

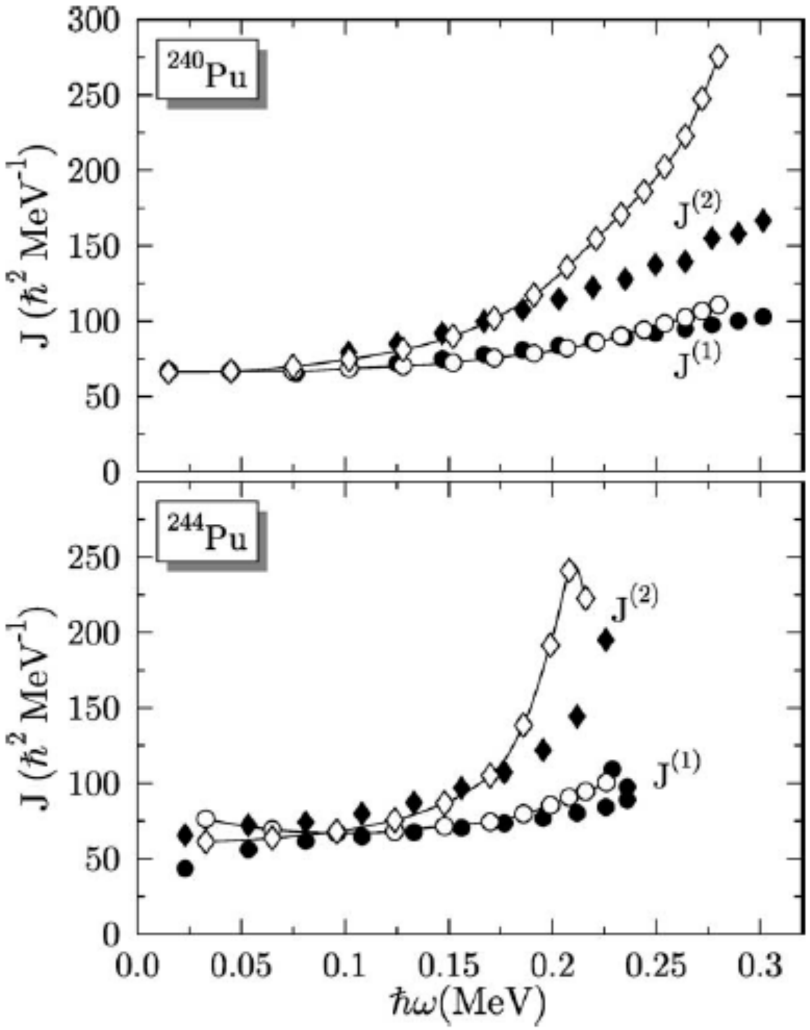


Fig. 3. Kinematical (circles) and dynamical (diamonds) moment of inertia for ^{240}Pu (top) and ^{244}Pu (bottom). Open (filled) markers denote calculated (experimental) values.

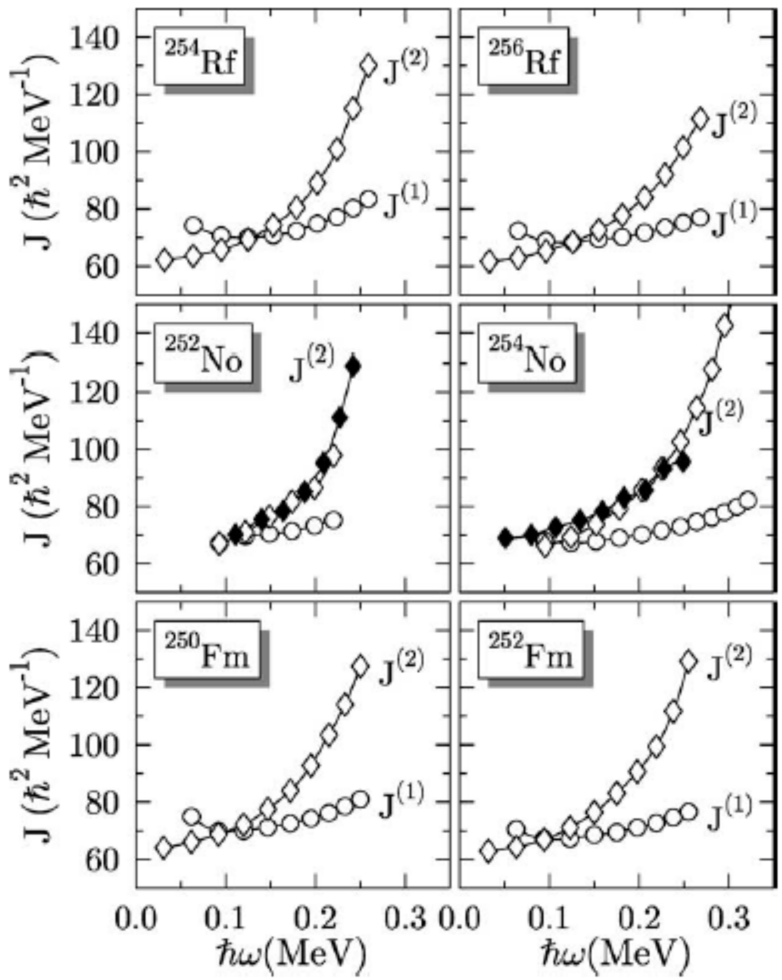
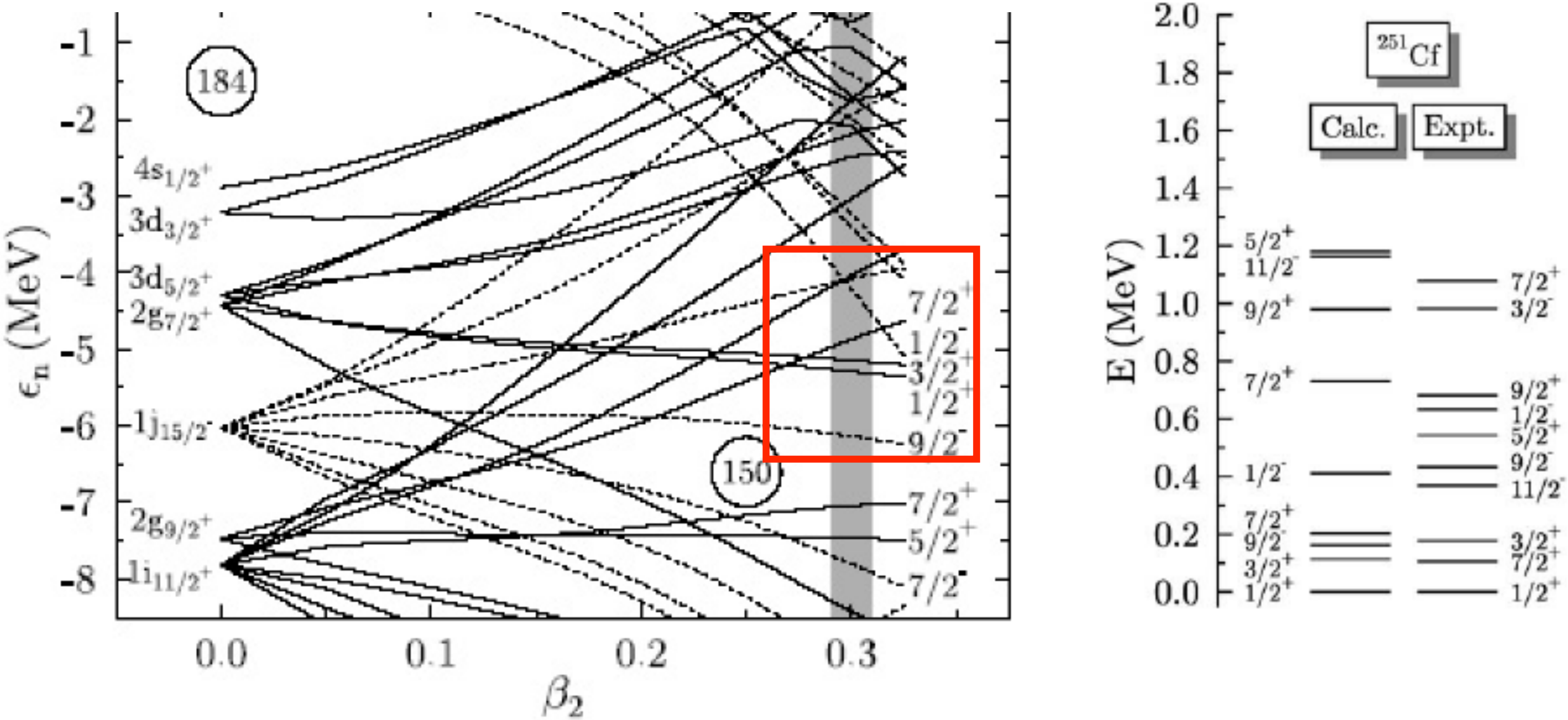


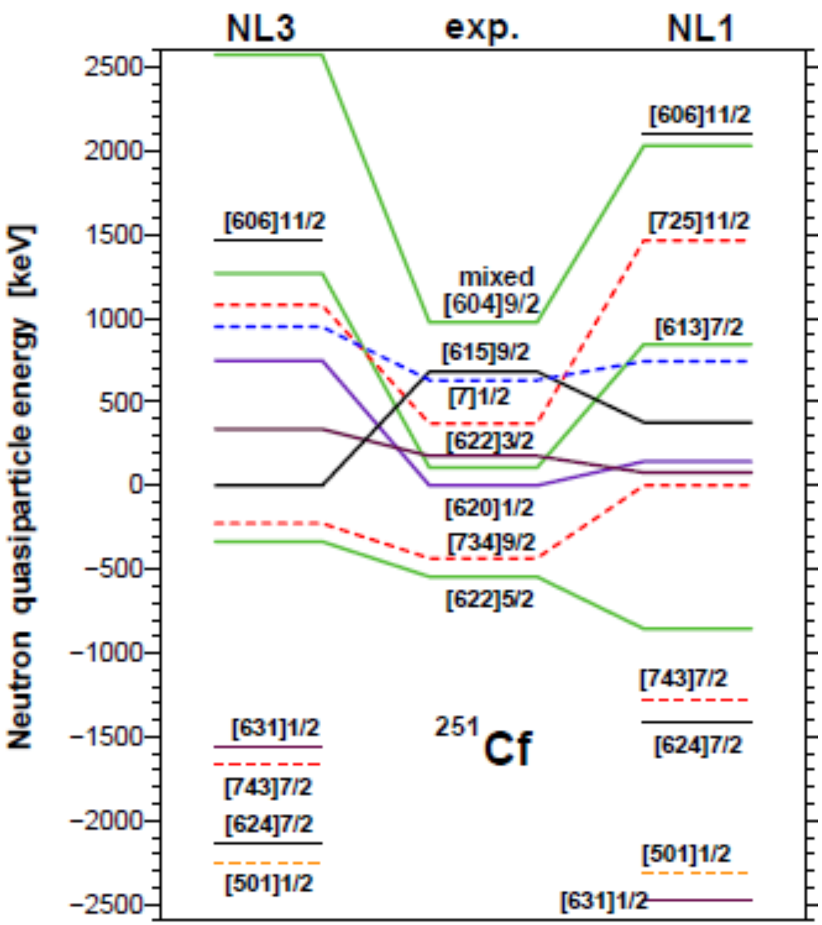
Fig. 5. Calculated kinematic, $J^{(1)}$, and dynamic, $J^{(2)}$, moments of inertia for nuclides with $Z = 100$–104 and $N = 150$, 152. Empty symbols are for calculations, full ones for experiment.

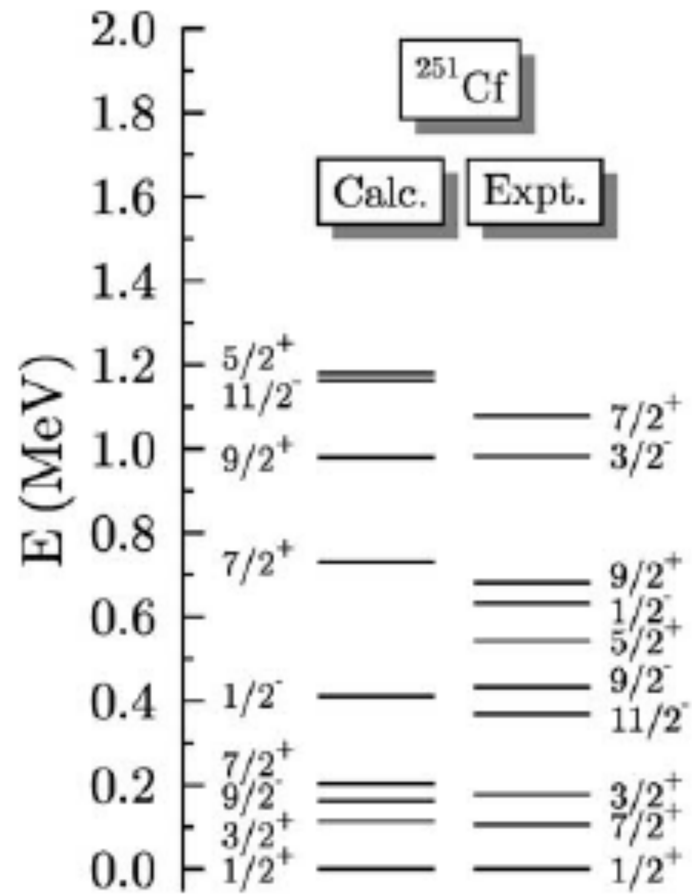
$$\beta_2^p = \sqrt{\frac{5}{16\pi}}\frac{4\pi}{3R^2Z}Q_2^p,$$

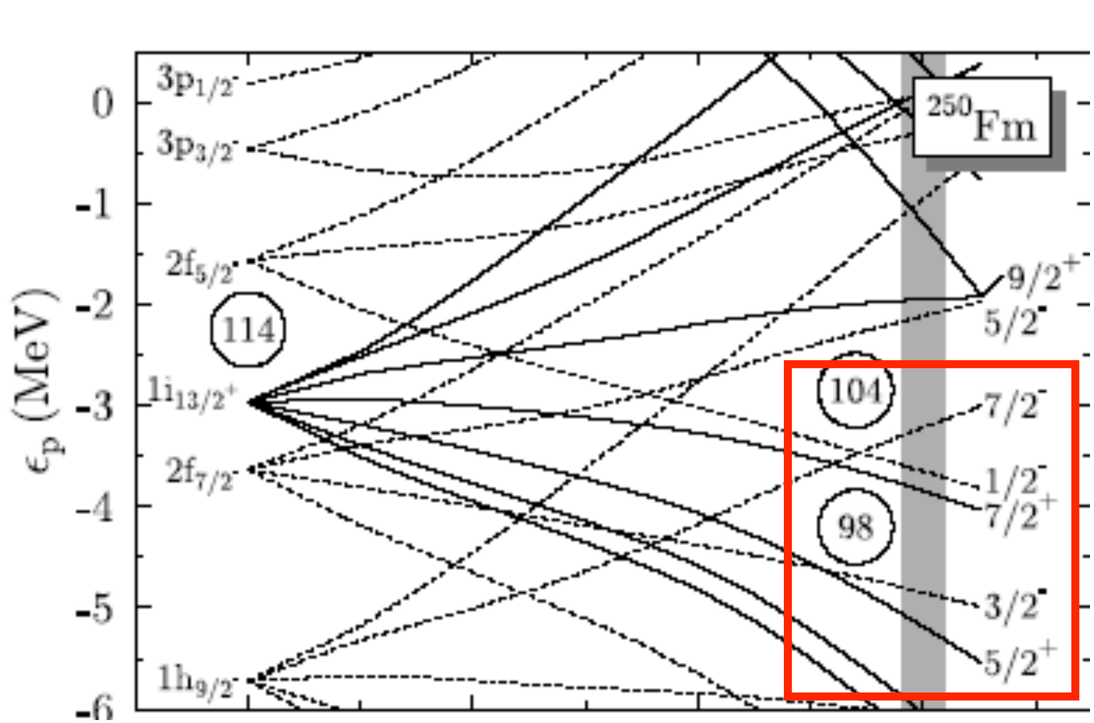


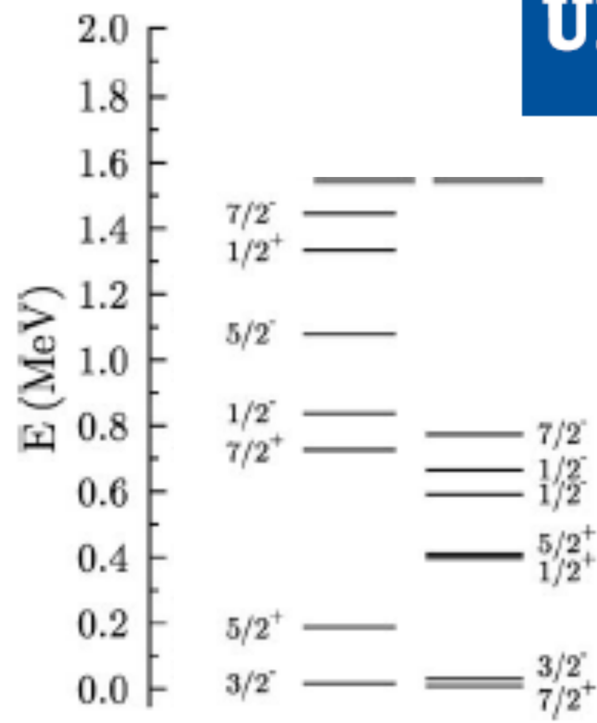
^{251}Cf   Z= 98   N=153

Too low 9/2$^+$ and too high 11/2$^+$: could be corrected by lowering 1j$_{15/2+}$


NL3
exp.
NL1
Neutron quasiparticle energy [keV]
2500
2000
1500
1000
500
0
-500
-1000
-1500
-2000
-2500
[606]11/2
mixed
[604]9/2
[615]9/2
[7]1/2
[622]3/2
[620]1/2
[734]9/2
[622]5/2
[725]11/2
[613]7/2
[631]1/2
[743]7/2
[624]7/2
[501]1/2
251Cf


251Cf
Calc.
Expt.
E (MeV)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
5/2+
11/2-
9/2+
7/2+
1/2-
7/2+
9/2-
3/2+
1/2+
7/2+
3/2-
9/2+
1/2-
5/2+
9/2-
11/2-
3/2+
7/2+
1/2+





^{249}Bk   N= 102 Z= 97

$5/2^+$ and $7/2^+$ improved if $j_{13/2+}$ lower

$½^-$ and $3/2^-$ closer if $f_{7/2-}$ $^{\text{and}}$ $f_{5/2-}$ closer

Effective mass of the EDF

Constrained to be around 0.7 m in SLyX EDF’s

Very low in RMF Lagrangians (around 0.6m)

An increase of the effective masse -> more dense spectrum
(based on Mahaux et al. in the 80’s , correlations increase the effective mass)

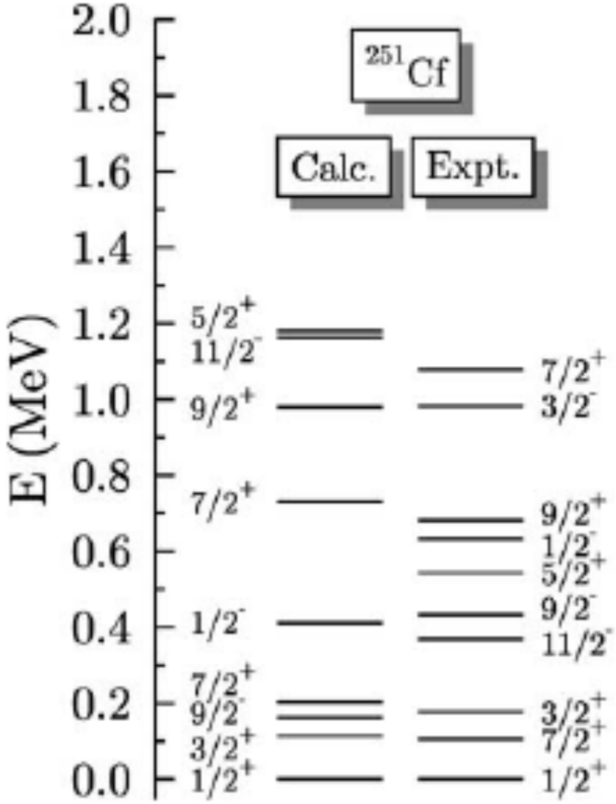
Pure WS -> effective mass = nucleon mass
(not the case in Chasman WS!)

Caution!

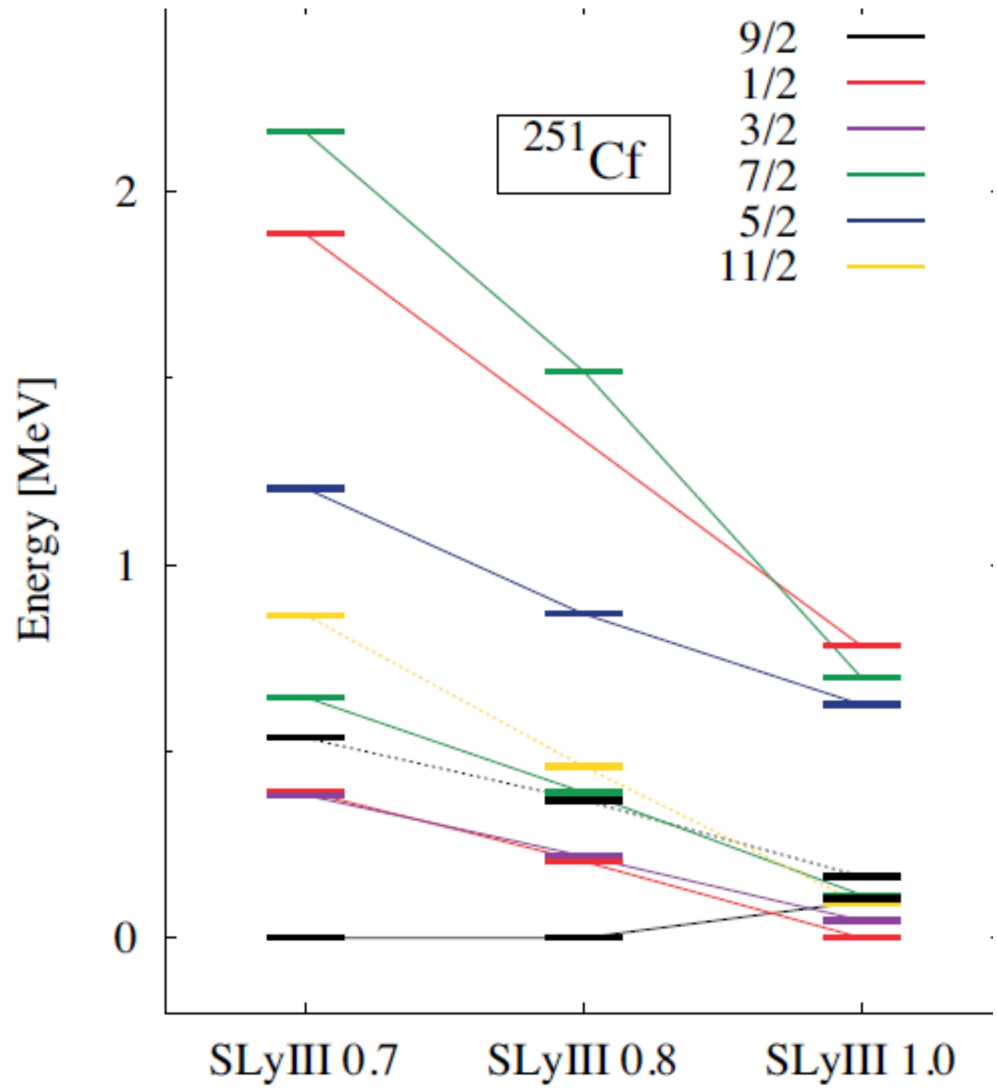
The effect of the effective can be drawn without ambiguity only
for parameterizations that have been adjusted in the same way.

SLIII family : improvement of SIII by a fitting protocol similar to the Sly's
- with inclusion of deformation properties of selected nuclei
- an exponent for the density dependence equal to 1
  (to avoid some problems in beyond mean-field calculations

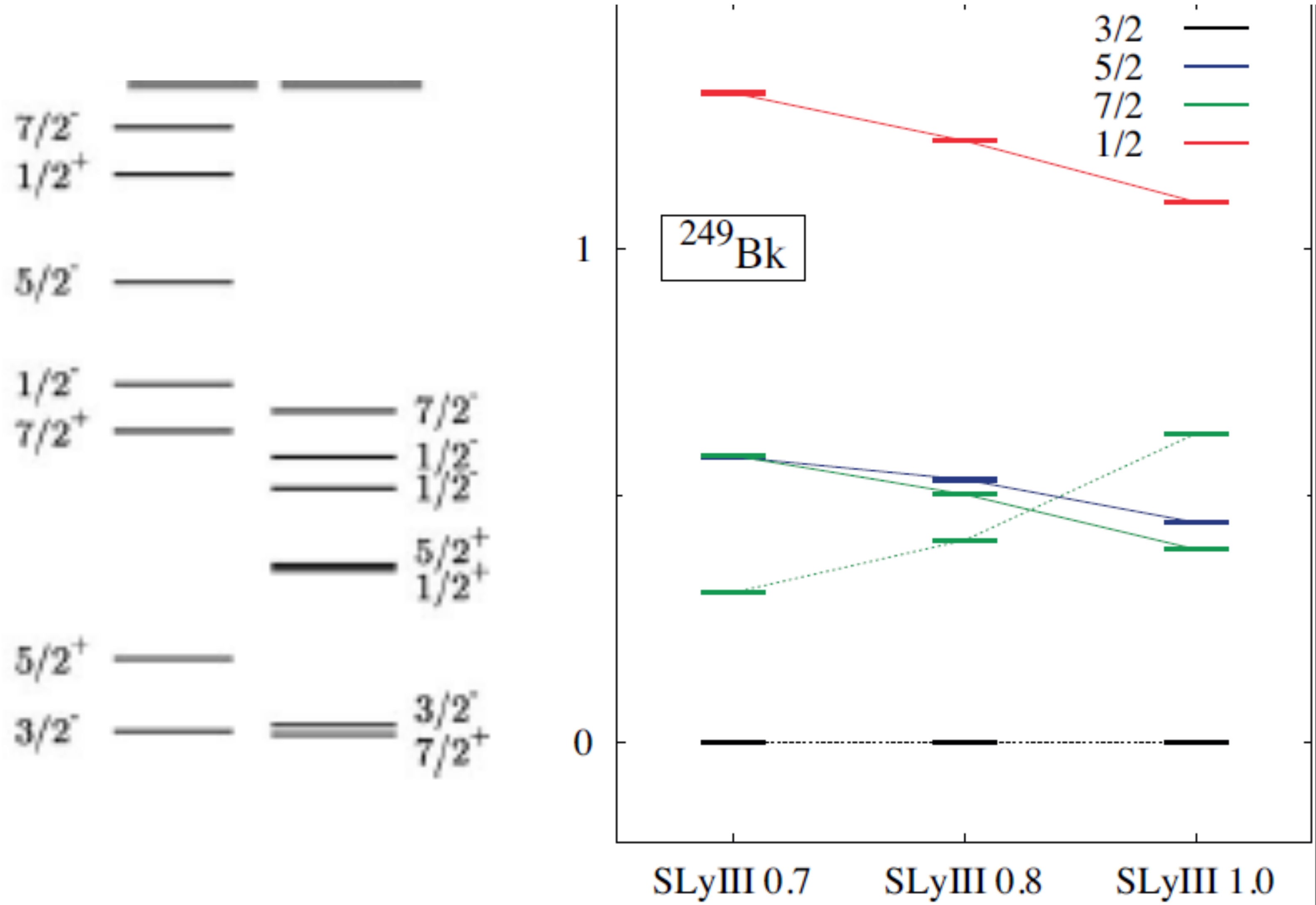
alpha = 1 -> compressibility of infinite nuclear matter cannot be right
effective mass can be varied (K and m^* strongly linked)



251Cf
Calc.
Expt.
E (MeV)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
5/2+
11/2-
9/2+
7/2+
1/2-
7/2+
9/2-
3/2+
1/2+
7/2+
3/2-
9/2+
1/2-
5/2+
9/2-
11/2-
3/2+
7/2+
1/2+



251Cf
Energy [MeV]
2
1
0
9/2
1/2
3/2
7/2
5/2
11/2
SLyIII 0.7
SLyIII 0.8
SLyIII 1.0



7/2⁻
1/2⁺
5/2⁻
1/2⁻
7/2⁺
5/2⁺
3/2⁻
7/2⁻
1/2⁻
1/2⁻
5/2⁺
1/2⁺
3/2⁻
7/2⁺
3/2
5/2
7/2
1/2
249Bk
1
0
SLyIII 0.7
SLyIII 0.8
SLyIII 1.0

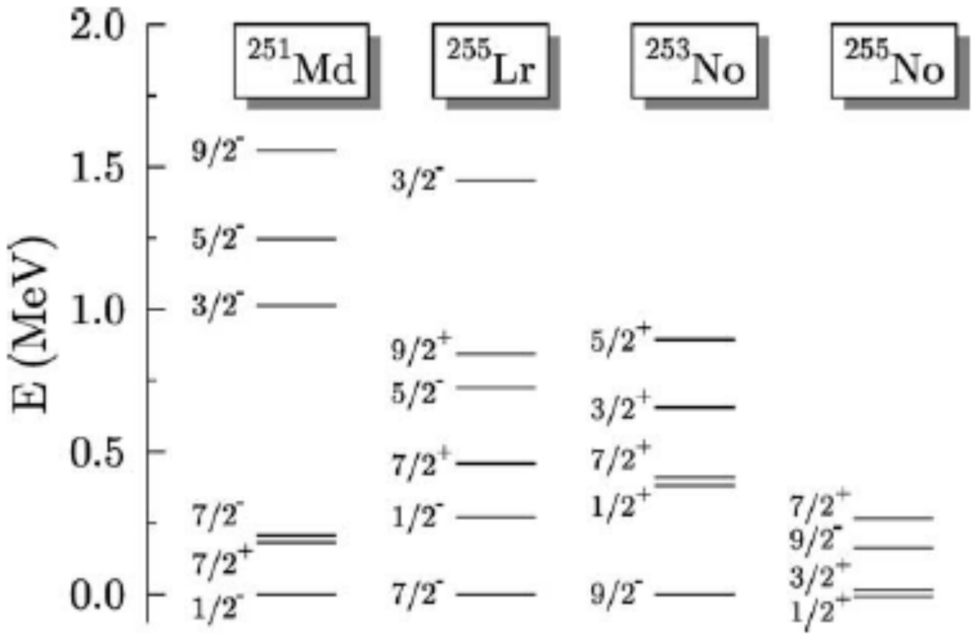
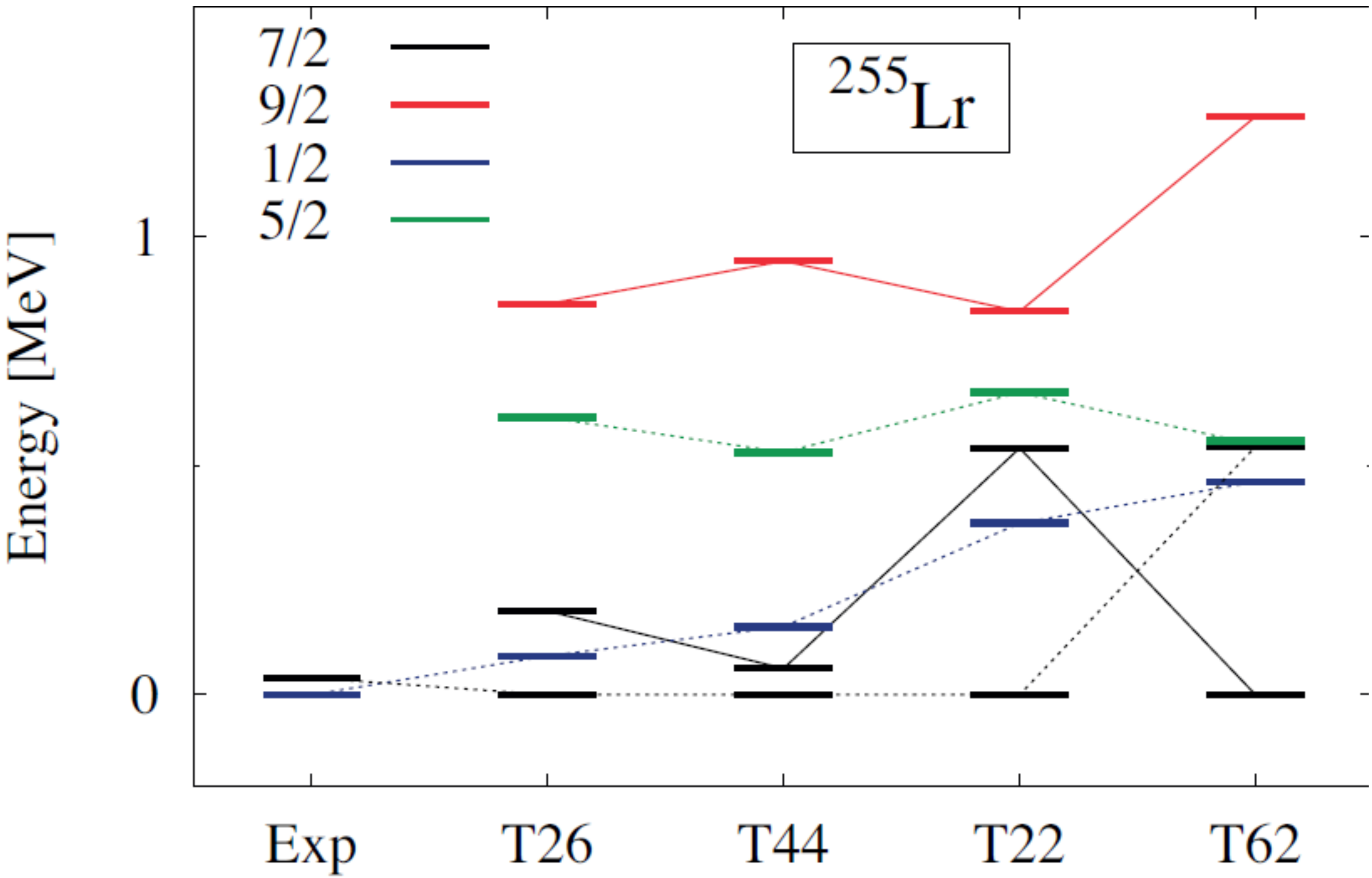
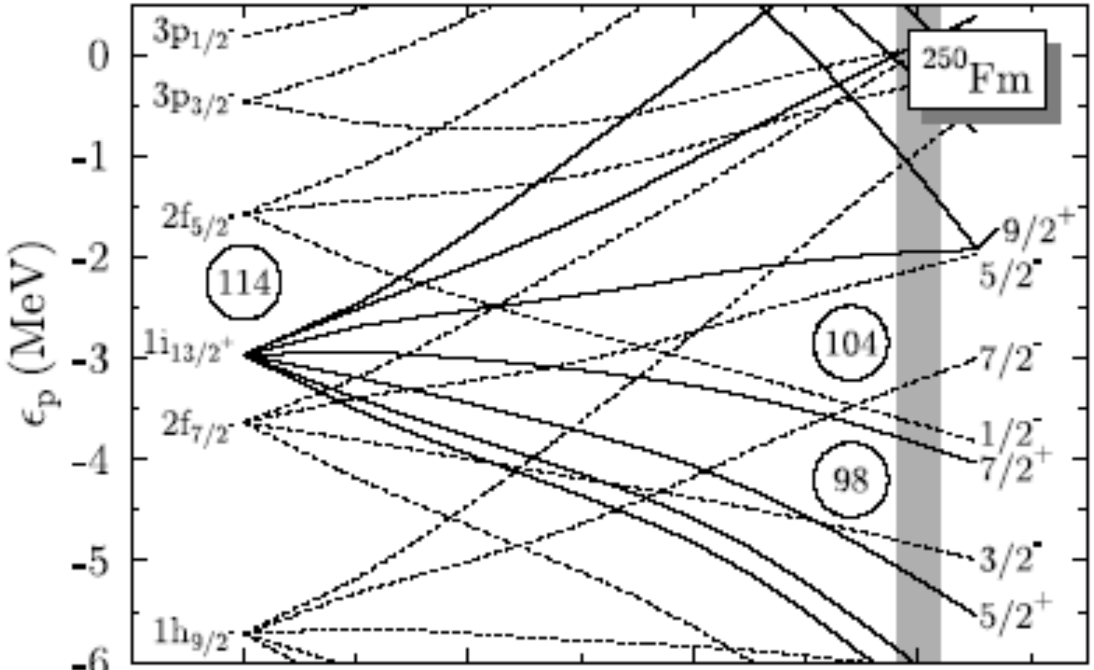


Fig. 4. Low lying energy spectra of ${}^{251}_{101}$Md, ${}^{255}_{103}$Lr (odd Z), ${}^{253}_{102}$No$_{151}$, and ${}^{255}_{102}$No$_{153}$ (odd N).

Has a tensor term in the EDF an effect on spectra?





Z = 99, 101, 103

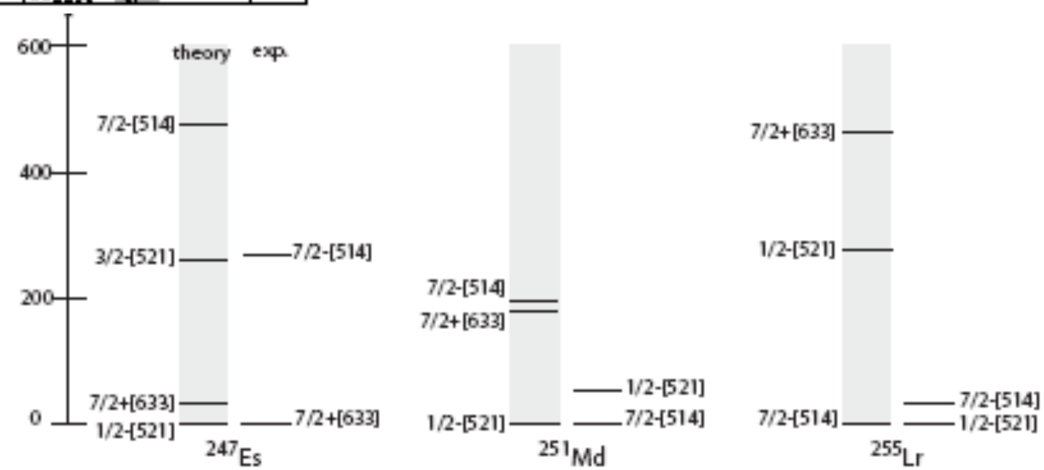


Fig. 12. Comparison between experimental and theoretical level schemes for ^{247}Es, ^{251}Md and ^{255}Lr.

Careful analysis of RMF results leads to similar conclusions: some orbitals are not placed as they should to reproduce experimental spectra with an accuracy better than 500 keV

The lowering of n $j_{15/2}$ and p i $_{13/2}$ can be done by increased spin orbit but not consistent with other analyses.

Changes pf spherical gaps would change the deformed Z=110 and Z=108 deformed gaps. Probably not much effects on Z=114

Conclusions

Full HFB calculations can be systematically done for SHE nuclei

Global properties (deformation, α decay energies, …) correctly described

Beyond mean-field methods should be applicable in a few years (projection, configuration mixing): particle-vibration coupling

The tool needed to improve the description of 1qp (and 2qp!) states has still to be found.